Emissivity Estimation Through the Solution of an Inverse Heat-Conduction Problem

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A new transient calorimetric method to measure the total hemispherical emissivity of opaque surfaces as a function of temperature is presented. A disk-shaped specimen for which the emissivity is required to be measured is placed in an evacuated vessel. One of its two surfaces is heated uniformly by means of an electrical resistance. The total hemispherical emissivity of the other—free—surface, which is parallel and very near to a uniformly cooled plate, can then be estimated using only transient temperature measurements at two locations inside the specimen. A model based on the solution of a one-dimensional inverse heat-conduction problem is used. This model allows one to estimate the surface temperature and net radiative heat flux from which the emissivity can be obtained. This property can also be estimated for paints and other surface coatings by covering the free surface with a thin layer of the coating concerned.

Nomenclature

A = sensitivity coefficient, Eq. (4)
B = Biot number
C = volumetric heat
e = emissivity error due to the solution algorithm
e' = emissivity error due to random measurement errors
E = root-mean-squared error based on e values, Eq. (14a)
E' = root-mean-squared error based on e' values, Eq. (14b)
F = number of future temperature measurements
h = radiative heat-transfer coefficient
k = thermal conductivity
ℓ = sample thickness
M = number of time steps
q = net radiative heat flux
t = time
T = temperature
U = Kirchhoff transformation, Eq. (7)
x = distance from the sample heated surface
Y = measured temperature
z = constant defined by Eq. (8e)
α = thermal diffusivity
γ = nonlinearity coefficient, Eq. (6)
Δt = time step size
Δx = space step size
ϵ = total hemispherical emissivity
λ = dummy variable, Eq. (9)
μ = constant of positive value, Eq. (5b)
σ = Stephan-Boltzmann constant, standard deviation
τ = calculation time, $= M\Delta t$
Φ = solution of Eq. (8) with $U(0,t) = U_{\max}$
Ψ = positive roots of Eq. (11)

Superscripts

j = time index such that $t = j\Delta t$
m = calculation instant index, $= 1,2,\ldots,M$

Subscripts

a = relative to the cooled surfaces
i = finite-difference point index
n = summation index
o = initial value
r = reduced value, as defined by Eq. (6d)
s = relative to the sample free surface ($x = \ell$)
$x1$ = relative to the first thermocouple location
$x2$ = relative to the second thermocouple location

Introduction

THE American space program of the 1960s created an important demand for accurate knowledge of thermophysical properties of many materials. Such knowledge is currently needed, not only for space programs based on industries of advanced technologies, but also for other progressing traditional industries. Some of these are the paints and other primary material transformation industries.

An important radiative property—even though global—is the total hemispherical emissivity, on which are based most industrial calculations. Calorimetric methods are essentially used to determine this property with the assumption of gray diffuse surfaces. Normally, a thin disk-shaped specimen is uniformly heated in an evacuated space to eliminate the convective losses. Heating can be carried out on a unique specimen whose back and lateral surfaces are guarded, or on two identical specimens placed around a heating plate (sandwhich form). In either permanent or transient modes, the emissivity is determined through a heat balance involving temperature measurement and heating power calculation. If it is easy to control the power and to measure the surrounding cooled surface temperature, it is, however, more difficult to measure precisely the specimen surface temperature. The transient calorimetric method presented here overcomes this difficulty by solving, in an original application, an inverse heat-conduction problem (IHCP).

The IHCP arises when direct measurement of surface conditions is not feasible. The surface temperature and heat flux may be estimated from internal temperature measurements. This problem has numerous applications in various branches of engineering and science. Some examples are the estimation of surface conditions in rocket nozzles, spacecrafts, combustion chambers, and nuclear reactors components. The IHCP arises also in geophysics, indirect calorimetry, quenching processes, melting, and ablation. An application introduced and examined in this paper deals—for the first time—with the estimation of total hemispherical emissivity of opaque surfaces and coatings.

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Many solutions for the IHCP were presented over the last three decades. These methods may be classified by their ability to treat nonlinear as well as linear cases. Numerical solutions based on finite differences or finite elements[1-6] have the inherent ability to treat nonlinear problems. Analytical methods[7-11] are usually restricted to linear cases. However, the analytical solution proposed by Imber and Khan[8] has been applied by Imber[10] to nonlinear problems through the use of an "equivalent diffusivity" over the whole time domain.

The IHCP is an ill-posed problem. Slight errors in interior measurements can magnify at the surface and lead to oscillations in the calculated surface conditions. Various corrective techniques were proposed to reduce such instabilities. These included the use of future temperatures associated with a least-squares method[1,2] or built-in smoothing,[3] regularized methods,[12] least-squares smoothing,[8-10] mollification or kernel method,[13,14] and the hyperbolic form of the heat-conduction equation.[4]

The availability of many efficient solutions for the IHCP led to the objective of this paper, which is to examine the feasiblity of applying one of these solutions to determine the total hemispherical emissivity of opaque materials as a function of temperature. However, the determination of this property is believed to be a parameter estimation problem.[18] So, the present method, although original and very simple, may not be the superior approach to treat our problem.

Estimation Model

For two opaque surfaces a and s, Fig. 1, supposedly gray, diffuse, and placed very near to each other in a vacuum environment, it is shown that[21]

$$\epsilon_s^m = q^m\{\sigma[(T_s^m)^4 - (T_a^m)^4] + q^m(1 - 1/\epsilon_a^m)\}^{-1} \qquad (1)$$

The cooled surface temperature T_a^m and emissivity ϵ_a^m are supposed to be known. The specimen surface temperature T_s^m and net radiative heat flux q^m are obtained by solving an IHCP using the data given by embedded thermocouples. Then, the transient estimation of the specimen surface emissivity ϵ_s^m is possible using Eq. (1).

Some comparisons[3,15,16] have shown that the solution of the IHCP of Beck et al.[2] is one of the most successful approaches currently in use. This method is therefore adopted in the present application. Possible general forms for this method are given in the literature.[16,17] The discretized one-dimensional scheme, which corresponds to our application, is shown on Fig. 1. It should be noted that the measuring points must coincide with finite-difference points.

Consider the region $x2 \le x \le \ell$. The data obtained from the second thermocouple, placed at $x = x2$, will be considered as a given boundary condition. Then, following the method proposed by Beck et al.,[2] the heat flux at the surface s is given by

$$q^m = q^{m-1} + \left[\sum_{j=m}^{m+F} (Y_{x1}^j - \tilde{T}_{x1}^j)\, A_{x1}^j \Big/ \sum_{j=m}^{m+F} (A_{x1}^j)^2\right] \qquad (2)$$

and the temperature field is computed from

$$T_i^m = \tilde{T}_i^m + (q^m - q^{m-1})\, A_i^m \qquad (3)$$

The $\tilde{T}_i^m$ are the temperatures obtained by solving a direct problem in the considered region with the unknown heat flux q^m replaced by the heat flux calculated at the last time step q^{m-1}. The A_i^m are the sensitivity coefficients given by

$$A_i^m = \partial T_i^m / \partial q^m \qquad (4)$$

and are obtained by differentiating the direct problem with respect to q^m and solving it as for $\tilde{T}_i^m$.

The problem is linearized at each time step by using thermal properties calculated at the last time step. An iterative procedure may be introduced to get more accurate results. This procedure converges rapidly, and it is not needed in linear cases. Detailed solutions are given in many references.[2,16,17]

Briefly, the net radiative heat flux q^m and surface temperature T_s^m are computed using Eqs. (2) and (3). Then, the total hemispherical emissivity ϵ_s^m is calculated using Eq. (1). Finally, the emissivity can be related to the transient temperature using a linear regression technique.

Test Problem

The following direct problem, which is very similar to the experimental situation, is used to simulate temperature measurements at the thermocouples locations. These measurements will be used to study the model performance in the next section.

Consider Fig. 1 with the cooled surface temperature T_a^m kept constant and equal to the specimen initial temperature T_o which in uniform. The problem may be formulated as follows:

$$\frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) = C \frac{\partial T}{\partial t} \qquad (5a)$$

$$T(0,t) = T_o + (T_{\max} - T_o)[1 - \exp(-\mu t)] \qquad (5b)$$

$$-k \frac{\partial T}{\partial x}\Big|_{x=\ell} = h\,(T_s - T_o) \qquad (5c)$$

$$T(x,0) = T_o \qquad (5d)$$

with

$$k(T) = k_o\,(1 + \gamma T_r) \qquad (6a)$$

$$C(T) = C_o\,(1 + \gamma T_r) \qquad (6b)$$

$$h(T_s) = h_o\,(1 + 0.5\gamma T_{rs}) \qquad (6c)$$

where

$$T_r = T - T_o, \qquad T_{rs} = T_s - T_o \qquad (6d)$$

The problem given by Eqs. (5) can be linearized by introducing the Kirchhoff transform $U(x,t)$ defined by[19]

$$U = \int_0^{T_r} (1 + \gamma T_r')\, dT_r' = T_r\,(1 + 0.5\gamma T_r) \qquad (7)$$

The problem becomes

$$k_o \frac{\partial^2 U}{\partial x^2} = C_o \frac{\partial U}{\partial t} \qquad (8a)$$

$$U(0,t) = \frac{(z-1)^2}{2\gamma}\exp(-2\mu t) - \frac{(z^2 - z)}{\gamma}\exp(-\mu t) + \frac{(z^2-1)}{2\gamma} \qquad (8b)$$

$$-k_o \frac{\partial U}{\partial x}\Big|_{x=\ell} = h_o U_s \qquad (8c)$$

$$U(x,0) = 0 \qquad (8d)$$

where

$$z = (1 + 2\gamma U_{\max})^{0.5} = \gamma(T_{\max} - T_o) + 1 \qquad (8e)$$

The linearized problem, Eqs. (8), can be solved by applying Duhamel's theorem:

$$U(x,t) = \int_0^t \frac{\partial U(0,\lambda)}{\partial \lambda}\, \Phi(x, t - \lambda)\, d\lambda \qquad (9)$$

where $\Phi(x,t)$ is the solution of the linearized problem with

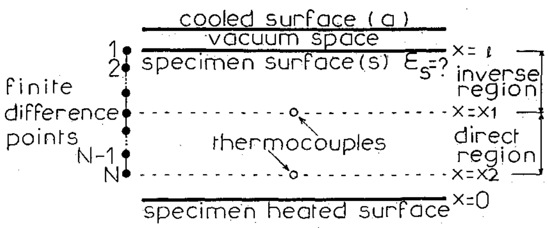

Fig. 1 Problem considered.

$\mu \to \infty$, i.e., with $U(o,t) = U_{max}$. This solution, given by Carslaw and Jaeger,[20] is as follows:

$$\Phi(x,t) = U_{max}\left[D(x) - \sum_{n=1}^{\infty} G_n(x)\exp(-\alpha\Psi_n^2 t/\ell^2)\right] \tag{10}$$

where Ψ_n are the positive roots of the transcendental equation

$$\Psi_n \cot\Psi_n = -B_o \tag{11}$$

The initial Biot number, B_o, is given by

$$B_o = h_o\ell/k_o$$

$$D(x) = \frac{1 + B_o(1 - x/\ell)}{1 + B_o}$$

$$G_n(x) = \frac{2(B_o^2 + \Psi_n^2)\sin(\Psi_n x/\ell)}{\Psi_n(B_o + B_o^2 + \Psi_n^2)}$$

The substitution of Eqs. (8b) and (10) into Eq. (9), then integrating,[16] leads to

$$U(x,t) = z(z-1)\left[D(x)P(x) - \sum_{n=1}^{\infty} G_n(x)V_n(t)\right] - (z-1)^2\left[D(x)R(t) - \sum_{n=1}^{\infty} G_n(x)W_n(t)\right] \tag{12}$$

$$P(t) = [1 - \exp(-\mu t)]/\gamma$$

$$R(t) = 0.5\,[1 - \exp(-2\mu t)]/\gamma$$

$$V_n(t) = \frac{[\exp(-\mu t) - \exp(-\alpha\Psi_n^2 t/\ell^2)]\,\mu\ell^2}{\gamma(\alpha\Psi_n^2 - \mu\ell^2)}$$

$$W_n(t) = \frac{[\exp(-2\mu t) - \exp(-\alpha\Psi_n^2 t/\ell^2)]\,\mu\ell^2}{\gamma(\alpha\Psi_n^2 - 2\mu\ell^2)}$$

Having calculated $U(x,t)$, the temperature is calculated from

$$T(x,t) = T_o + [(1 + 2\gamma U)^{0.5} - 1]/\gamma$$

For linear cases, ($\gamma = 0$): $T(x,t) = U(x,t) + T_o$. Noting Eqs. (1), (5c), and (6c), the emissivity—which is necessarily a function of the surface temperature T_s—is given by[16]

$$\epsilon_s = \frac{h_o T_{rs}(2 + \gamma T_{rs})}{2\ell\sigma(T_s^4 - T_o^4) + h_o T_{rs}(2 + \gamma T_{rs})(1 - 1/\epsilon_a)} \tag{13}$$

It should be noted that T_s, which is calculated from Eq. (12), and T_o are absolute values in Eq. (13).

Performance Study

The accuracy of the estimation model and its sensitivity to various errors are briefly studied in this section. Exact emissivity and temperature measurements at $x1$ and $x2$ are simulated using Eqs. (12) and (13) with the following data:

$T_o = 283$ K $\quad T_{max} = 333$ K

$k_o = 0.2117$ W/mK $\quad C_o = 1633172$ J/m³K

$\gamma = 0.0032\ \mathrm{K}^{-1}$ $\quad \mu = 0.0005\ \mathrm{s}^{-1}$

$\ell = 0.008$ m $\quad x = x1 = 0.007$ m

$\tau = 9000$ s

$h_o = 5.2925$ W/m²K

$\epsilon_a = 0.95$

$x = x2 = 0.0015$ m

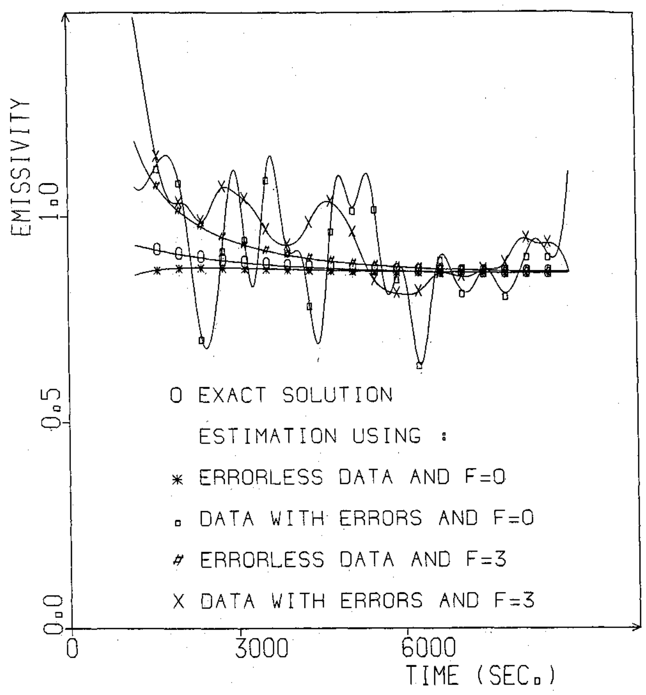

Fig. 2 Theoretical results.

The corresponding Biot number B_o is then 0.2. Under these conditions, the emissivity was found to vary from 0.976 (at $T_s = 293$ K) to 0.863 (at $T_s = 326.23$ K). The erroneous temperature measurements are simulated by adding independent random errors of zero mean and constant variance of 0.3 to the exact measurements. The standard deviation corresponds approximately to 1% of the maximum temperature. The exact and inexact measurements are then used to estimate the emissivity through Eqs. (1-3) with $\Delta x = 0.0005$ m, $\Delta t = 300$ s, and $F = 0$ and 3. The results are shown in Fig. 2, in which the accuracy and the sensitivity to random measurement errors may be characterized by

$$E = \left[\frac{1}{M}\sum_{m=1}^{M}(e^m)^2\right]^{0.5} \tag{14a}$$

as a measure for the accuracy, and

$$E' = \left[\frac{1}{M}\sum_{m=1}^{M}(e'^m)^2\right]^{0.5} \tag{14b}$$

as a measure for the sensitivity to random measurement errors. Note that e is the difference between the exact emissivity and the emissivity estimated using exact measurements, and e'

is the difference between the emissivity estimated from exact and inexact measurements.

Low values of E and E' mean, and respectively, a highly accurate and less sensitive solution. However, the highest emissivity errors are noted to be at the early time steps. So, regardless of the emissivity values estimated at $m = 1$, we have the following:

For $F = 0$:

$$E = 0.02, \qquad E' = 0.19$$

For $F = 3$:

$$E = 0.20, \qquad E' = 0.09$$

As noted by other investigators,[15] a higher stability is achieved along with a loss of accuracy. In the present application, this is done by increasing F. However, since the emissivity is a bounded parameter ($0 \leq \epsilon \leq 1$), the E and E' values given previously become smaller if the "bad values," i.e., $\epsilon < 0$ or $\epsilon > 1$, are omitted.

The effect of random measurement errors associated with the cold surface temperature was examined by imposing a bounded random error of $\pm 1\%$ on T_a. The corresponding emissivity error was found to be less than 0.5%. Noting that this temperature plays no role in the extrapolation algorithm and that the temperatures simulated inside the solid are exact, the increase of F leads to some additional inaccuracies.

On the other hand, an error of 5% on the cold surface emissivity has led to an error of the same order ($\simeq 5\%$) on the estimated specimen surface emissivity.

The effect of uncertainties associated with the thermal properties seems to be dominated by the error of the thermal conductivity k. An error of 2.5% on this property has led to an error of about 5% on the emissivity.

Inaccurate estimations may also arise from thermocouple calibration and positioning errors. These are simulated by adding 2.5% errors to each parameter, i.e., T_{x1}, T_{x2}, $x1$, and $x2$. The resulting emissivity errors were found to be about $\pm 5.5\%$ and $\pm 3\%$ in case of calibration and positioning errors, respectively.

A more detailed study is given by Osman.[16] However, most of these errors can be easily brought to reasonable levels in practical applications.

Experimental Setup

The setup, which is placed in an evacuated vessel, is shown in Fig. 3. It comprises the following:

1) A copper cylinder whose down surface is uniformly cooled with water circulation. This surface is parallel and very near to the specimen upper surface. In addition, it is covered with a very thin layer of black paint whose emissivity is known. 2) A uniform heating system composed of a spiral resistance and a copper plate. 3) Tight thermocouple and electric passages. 4) A plexiglass specimen whose upper surface is covered with a very thin layer of the concerned paint.

The use of a very thin layer of the paint should be emphasized. In this case, the paint temperature can be well approximated by the plexiglass surface temperature. If the paint thermal resistance is high, e.g., due to a considerable thickness, then a IHCP in a composite media should be solved.

The plexiglass thermal properties in the temperature range 20–60 °C, given by Raynaud,[22] are $k = 0.2117\ [1 + 0.0032\ (T - 20]$ W/mK, and $C = 1739000$ J/m^{3}K. No error limits are given for these values.

The specimen dimensions and thermocouples locations are shown in Fig. 4. Chromel/alumel theromcouples of 0.08 mm diam are used. The ratio D/d allows one to adopt a one-dimensional analysis of heat flow inside the specimen, which is heated uniformly. The maximum heating temperature is 60 °C (333 K). At this temperature, 95% of the total emissive power lies in the wavelength range 5–45 μm. For a plexiglass specimen of 1 mm depth, the transmittivity is nearly zero beyond the wavelength 5 μm. Thus, the specimen is practically opaque to thermal radiation. However, the plexiglass was used for the relative facility of placing the thermocouples inside it and then measuring their real positions as accurately as possible. The technique is to mold the plexiglass around the thermocouples and then to measure their real positions using x rays. The positions were found to be 0.98 and 1.03 mm from each surface, and thus are approximated by 1 mm as shown in Fig. 4. These positions satisfy the condition of opacity and guarantee a reasonable performance of the model.

Experimental Results

The emissivity was estimated for three paints: 1) an ivory colored aerosol paint; 2) a metallic, silver colored, aerosol paint; and 3) an aerosol black paint (mat).

The estimation procedure is as follows:

1) Estimate the paint temperature and the corresponding emissivity using Eqs. (1–3).

2) Eliminate the estimated "bad values" ($\epsilon < 0$ or $\epsilon > 1$).

3) Relate the estimated "good values" ($0 \leq \epsilon \leq 1$) to the corresponding temperatures using a linear regression technique.

4) Estimate the mean value ϵ_m and the standard deviation σ_ϵ.

By increasing F and repeating the above procedure, the mean value and standard deviation can change. The case with a decreasing and a fairly unchanged ϵ_m can be adopted. Such a stable scheme may be also obtained with no need to increase F, due to the elimination of "bad" emissivity values. However, as a result of the central limit theorem, a confidence

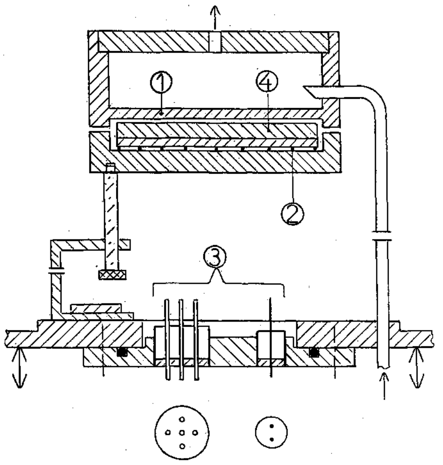

Fig. 3 Experimental setup.

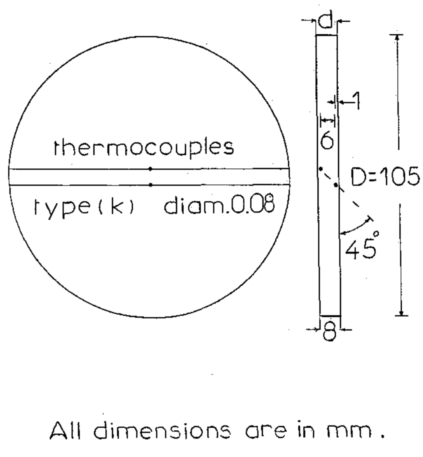

Fig. 4 Specimen.

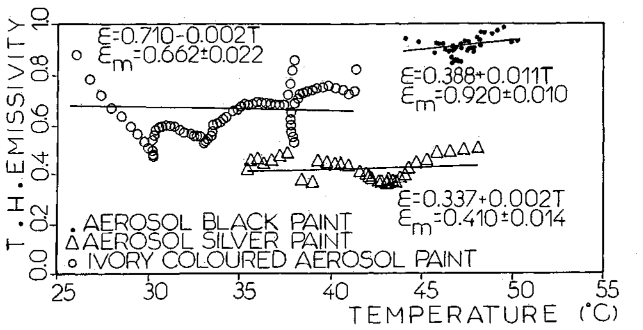

Fig. 5 Experimental results.

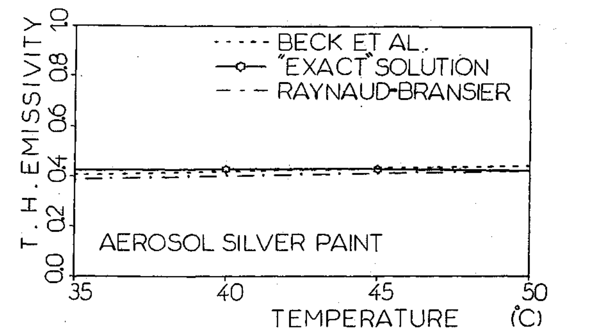

Fig. 6 Comparison of results.

bound on ϵ_m for a given probability can be estimated. For each paint, the confidence bound is given here using a 95% probability, as shown in Fig. 5.

Theoretical Verification of Results

Because of the lack of precise information about the emissivity of paints under variable temperature conditions, we tend here to verify our results theoretically using two methods.

One method is to use the exact solution, presented here, to reestimate the emissivity using the real measurements. The procedure is very simple and can be summarized as follows:

1) Reformulate the measured temperature $Y_{x2}(t)$ so that

$$Y_{x2}(t) \simeq T_{x2}(t) = T_o + (T_{\max} - T_o)[1 - \exp(-\mu t)] \quad (15)$$

where $T_{\max}$ are to be estimated using the differential correction method.

2) Approximate the volumetric heat C by the relation

$$C \simeq C_o\,(1 + \gamma T_r) \quad (16)$$

where $T_r = T - T_o$. The thermal conductivity is already given under the same form. Therefore, using the same γ the value of C_o is chosen such that the real C—which is constant—represents a mean value in the same temperature range.

3) Estimate the unknown initial heat-transfer coefficient h_o which better fits the experimental data. This can be done graphically by substituting h_o values in Eq. (12) and calculating, for each value, the corresponding function $S(h_o)$ defined by

$$S(h_o) = \sum_{m=1}^{M} |T_{x1}^m(h_o) - Y_{x1}^m| \quad (17)$$

The value of h_o which gives a minimum $S(h_o)$ is selected. This can be easily done while noting that $h_o < 6$, according to Eq. (13). A quasisimilar approach to estimate the heat-transfer coefficient iteratively in linear problems is given by Mehta.[11]

It should be noted that the exact solution satisfies Eq. (6c). For a limited temperature range, this equation can be considered as a reasonable approximation which may be slightly different from the real $h(T_s)$ variation.

The knowledge of $T_{\max}$, γ, μ, and h_o finally allows the estimation of surface temperatures and the corresponding emissivities using Eqs. (12) and (13).

However, the exact solution can be applied only if the cold surface temperature is constant and equal to the specimen intial temperature. During the experiments, the cold surface temperature was varying considerably except for the metallic paint, probably due to its relatively low emissivity. This means that the exact solution can be applied only for this paint, where the cold surface temperature increased only from 19.5-20.5°C. A mean value of 20°C is therefore used. The result is shown in Fig. 6. It can be seen that the estimated heat-transfer coefficient h_o—which is found to be 2.873 W/m^{2}K, i.e., $B_o = 0.095$, in the range 20-50°C—has led within the same temperature range to values for ϵ very near to the mean value obtained experimentally using Beck's method.

Another efficient solution for the IHCP is also used to confirm this result. The space-marching method developed by Raynaud and Bransier[3] is applied to estimate the temperature field in the inverse region, Fig. 1. This is done in two steps:

Step 1:

$$T_{i-1}^{m+1} = \left(\frac{\omega+1}{\xi}\right) T_i^{m+2} + \left(\frac{\xi-1}{\xi}\right) T_i^{m+1} - \left(\frac{\omega}{\xi}\right) T_{i+1}^{m+2} \quad (18)$$

where

$$\xi = \frac{k_{i-1}^{m+1} + k_i^{m+1}}{C_i^{m+1} + C_i^{m+2}} \frac{\Delta t}{(\Delta x)^2}, \qquad \omega = \frac{k_i^{m+2} + k_{i+1}^{m+2}}{C_i^{m+1} + C_i^{m+2}} \frac{\Delta t}{(\Delta x)^2}$$

Step 2:

$$T_{i-1}^{m+1} = \left(\frac{\eta+1}{\eta}\right) T_i^{m+1} + \left(\frac{\nu-1}{\eta}\right) T_i^{m} - \left(\frac{\nu}{\eta}\right) T_{i+1}^{m} \quad (19)$$

where

$$\eta = \frac{k_{i-1}^{m+1} + k_i^{m+1}}{C_i^{m} + C_i^{m+1}} \frac{\Delta t}{(\Delta x)^2}, \qquad \omega = \frac{k_i^{m} + k_{i+1}^{m}}{C_i^{m} + C_i^{m+1}} \frac{\Delta t}{(\Delta x)^2}$$

The final temperature is the arithmetic mean of the two estimators. The heat flux is calculated from a heat balance at the surface s. The emissivity is then calculated using Eq. (1) and is related to the temperature using a linear regression technique. The result is summarized in Fig. 6. More details are given by Osman.[16]

It can be seen that the total hemispherical emissivity obtained by this method is very near to that obtained by the other two methods. This confirms the validity of the proposed technique.

Conclusions

A new transient calorimetric method to estimate the total hemispherical emissivity of opaque materials, based on the solution of an inverse heat-conduction problem, was presented in this paper. The method was theoretically tested to find the best operating conditions and then applied experimentally to three different paints. The results were shown to be reasonable and reproducible. The validity was proven by comparing the results obtained through the use of three different methods.

A plexiglass specimen was used due to the relative facility of manipulating this material and placing the thermocouples inside it with high precision. In such a material, the temperature domain should be limited so that the assumption of opacity could be valid, and also due to the low fusion temperature of this material. However, the emissivity could have been obtained using a metallic specimen which offers the possibility of increasing the temperature domain. This is a next step in this work. The results will be compared with those obtained through other methods, which are currently under development in our laboratory.

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